A Method for the Direct Synthesis of General Sections

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***Abstract*—A network synthesis method is presented that permits the direct synthesis of an arbitrary number of in-line General Sections from the filter impedance function.**

I. Introduction

The General Section is a cross-coupled filter structure that is widely used in microwave filter topologies to realize two finite frequency attenuation poles. The coupling arrangement for a General Section is shown in Fig. 1.

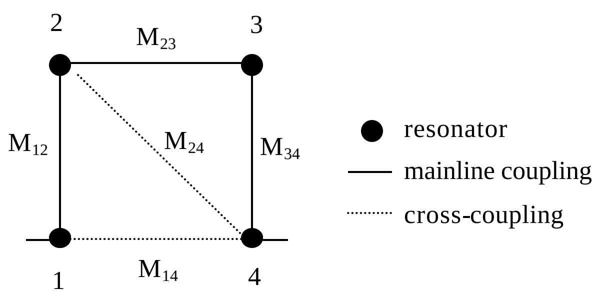

Fig. 1. General Section

The General Section may be incorporated into both folded and in-line filter structures, as shown in Fig. 2.

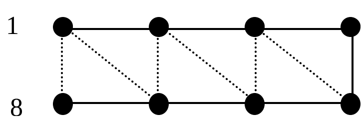

Fig. 2a. Folded structure

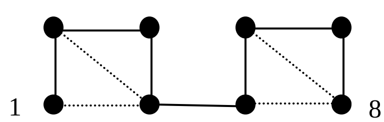

Fig. 2b. In-line structure

In a folded structure, the elements of multiple General Sections can be evaluated using the method of turning, described by Cameron [1]. However, because of topological constraints, it is not possible to extract more than one General Section from an in-line network using this method.

Previous solutions for extracting in-line filter networks containing General Sections have involved the application of matrix rotation techniques to the network's canonical form. However, as pointed out by Levy [2], this is a rather awkward process and has not been developed beyond even order 10.

In this paper, an equivalent lumped element extraction of a General Section is presented. The extracted network elements can be transformed directly into the elements of the corresponding General Section using the relationships described in Section III. There is no inherent limitation on the network order nor on the number of General Sections that may be extracted.

The authors have exercised the procedure exhaustively, and have successfully applied the technique to all four classes of General Section. As additional confirmation of the validity of the procedure, the technique automatically reduces to the cascaded quadruplet (CQ) case described by Levy [2] when symmetric real or imaginary axis poles are specified.

II. Direct Synthesis of the General Section

In Fig. 1, each of the resonator nodes consists of a shunt capacitor and shunt frequency invariant susceptance connected to ground. In a similar fashion to the Cascaded Quadruplet (CQ) section described by Levy [2], the circuit elements at node 1 may be extracted prior to the General Section extraction. The circuit elements at node 4 may be extracted after the General Section has been removed from the network. The remaining partial General Section is shown in Fig. 3.

In order to verify that the General Section is extractable from the overall network transfer function, the admittance matrix of the partial General Section will be analyzed.

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In Fig. 3, the lines joining the nodes are admittance inverters. Inverters M_{12} and M_{34} may be set to unity admittance without loss of generality.

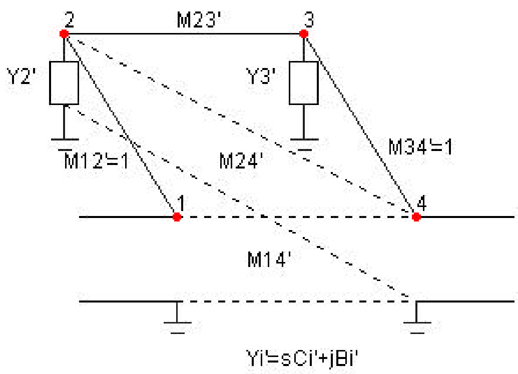


Fig. 3. Partial General Section

The admittance matrix of the partial General Section is given by

$$[Y] = K_1 \begin{bmatrix} Y_{11} & Y_{14} \\ Y_{41} & Y_{44} \end{bmatrix} \tag{1}$$

where

$$K_1 = \frac{1}{\left(M_{23}^2 - Y_2 Y_3\right)} \tag{2}$$

$$Y_{11} = jY_3 \tag{3}$$

$$Y_{14} = Y_{41} = -j\left(M_{23} - M_{24}Y_3 - M_{14}\left(M_{23}^2 - Y_2Y_3\right)\right) \tag{4}$$

$$Y_{44} = \frac{-j\left(M_{23}^2 - Y_2Y_3 - \left(M_{23} - M_{24}Y_3\right)^2\right)}{Y_3} \tag{5}$$

$$Y_2 = sC_2 + jB_2 \tag{6}$$

$$Y_3 = sC_3 + jB_3 \tag{7}.$$

Admittances Y_{14} and Y_{41} are second order functions of w. Each of these admittances is equal to zero at frequencies w_1, w_2, the attenuation poles of the General Section, and are given by

$$w_{1,2} = \frac{1}{2K_2}\left(-(K_3) \pm \sqrt{(K_3)^2 + 4(K_4)}\right) \tag{8}$$

where

$$K_2 = M_{14}C_2C_3 \tag{9}$$

$$K_3 = \left(C_3\left(M_{14}B_2 - M_{24}\right) + B_3M_{14}C_2\right) \tag{10}$$

$$K_4 = \left(M_{24}B_3 - M_{23}\left(M_{14}M_{23} - 1\right)\right)M_{14}C_2C_3 \tag{11}.$$

Thus at frequencies w_1, w_2 there is no transmission. This verifies that the General Section's partial transfer matrix, given below in (12), is extractable from the overall transfer matrix of a filter network [3].

The removal of the partial General Section from the overall network can be expressed in terms of the chain matrix as

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \frac{1}{K_5}\begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \tag{12}$$

where

$$K_5 = \left(1 - M_{14}M_{23}\right)\left(1 - \frac{M_{14}Y_2Y_3}{M_{23}\left(M_{14}M_{23} - 1\right)}\left(1 - \frac{M_{24}}{M_{14}Y_2}\right)\right) \tag{13}$$

$$A' = \frac{-\left(M_{23}^2 - Y_2Y_3 - \left(M_{23} - M_{24}Y_3\right)^2\right)}{M_{23}Y_3} \tag{14}$$

$$B' = j\left(\frac{Y_2Y_3}{M_{23}} - M_{23}\right) \tag{15}$$

$$C' = -j\frac{\left(K_6 + K_7\right)}{M_{23}\left(M_{23}^2 - Y_2Y_3\right)} \tag{16}$$

where

$$K_6 = \left(M_{23}^2 - Y_2Y_3 - \left(M_{23} - M_{24}Y_3\right)^2\right) \tag{17}$$

$$K_7 = M_{23} - M_{24}Y_3 - M_{14}\left(M_{23}^2 - Y_2Y_3\right)^2 \tag{18}$$

and

$$D' = \frac{Y_3}{K_{23}} \tag{19}.$$

Equations (8) and (12-14) reduce to the same form as those of Levy's transfer matrix for the partial CQ when M_{24}, B_2, B_3 are simultaneously set to zero, i.e. $M_{24} = B_2 = B_3 = 0$.

III. NETWORK TRANSFORMATION

In the previous Section the existence of the General Section synthesis was demonstrated. Rather than writing a special synthesis program, it is more convenient to transform cascaded lowpass filters of defined topology into the General Section format using a circuit transformation. The lowpass prototype network for a complete General Section is shown in Fig. 4.

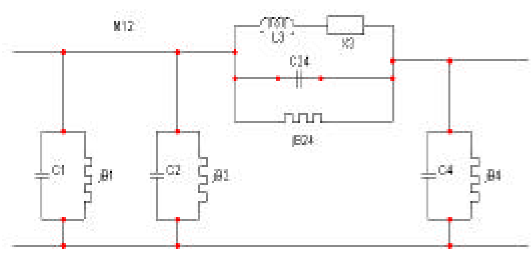


Fig. 4. Lowpass prototype of the General Section

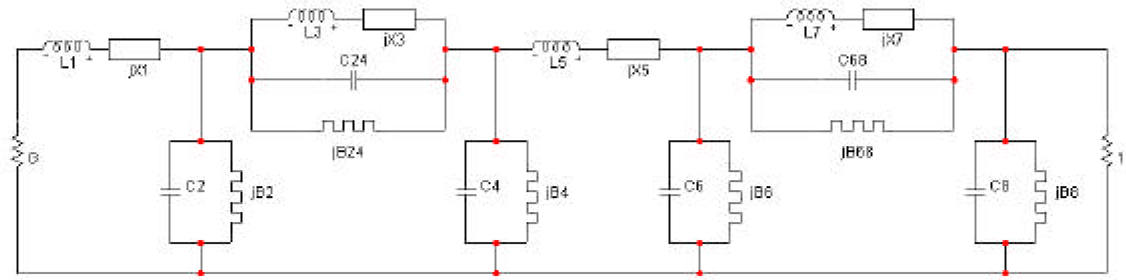
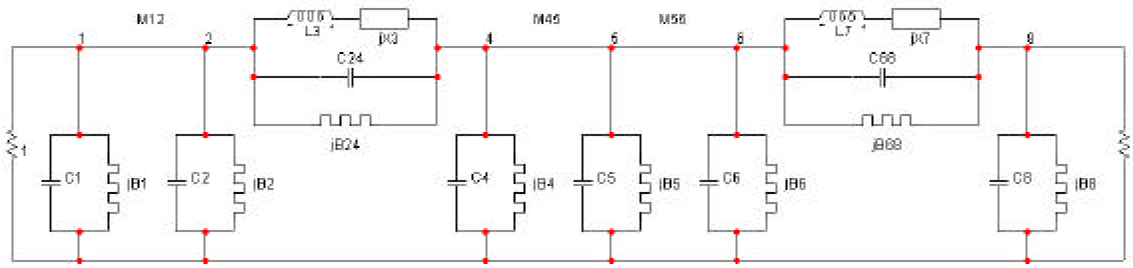


Fig. 5a. Lowpass prototype filter



$C_1 = M_{12}^2 L_1$ $\quad B_1 = M_{12}^2 X_1$ $\quad M_{12} = \sqrt{G}$ $\quad M_{45} = 1$ $\quad M_{56} = 1$

Fig. 5b First stage of transformation

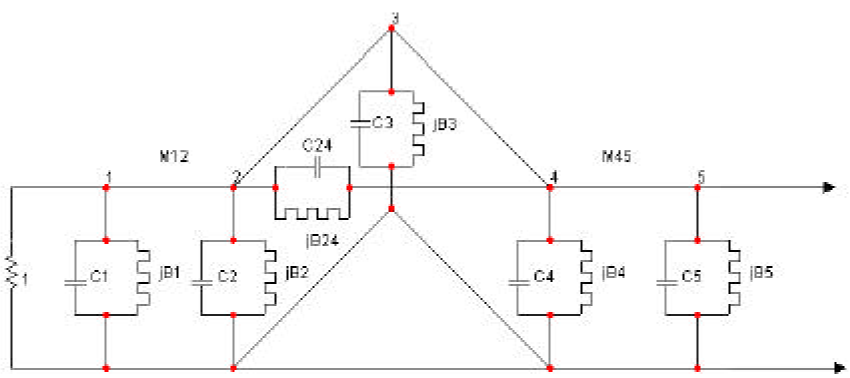
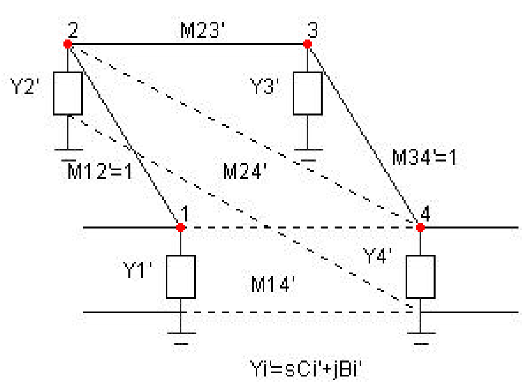


Fig. 5c Second stage of transformation



$$Y_1' = (sC_1 + jB_1) \qquad Y_2' = \frac{s(C_2 + C_{24}) + j(B_2 + B_{24})}{M_{12}^2}$$

$$Y_3' = \left(\frac{C_2 + C_{24}}{C_2}\right)^2 (sC_3 + jB_3) \qquad C_3 = L_3 \qquad B_3 = X_3$$

$$Y_4' = s\left(C_4 + \frac{C_2 C_{24}}{C_2 + C_{24}}\right) + j\left(B_4 + B_{24}\left(\frac{C_2}{C_2 + C_{24}}\right)^2 + B_2\left(\frac{C_{24}}{C_2 + C_{24}}\right)^2\right)$$

$$M_{23}' = \left(\frac{C_2 + C_{24}}{C_2 M_{12}}\right) \qquad M_{14}' = \frac{-C_{24} M_{12}}{C_2 + C_{24}}$$

$$M_{24}' = \left(\frac{C_2 B_{24} - B_2 C_{24}}{M_{12}(C_2 + C_{24})}\right)$$

Fig. 5d Final conversion

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The eighth degree filter shown in Fig. 5a will be used as an example for the transformation. This filter has a fourth ordered attenuation pole at infinity and a pair of second order finite poles, giving a total filter degree of 8.

The first step in the transformation of this circuit into a General Section network topology is to replace each series inductor and series frequency invariant reactance by a cascade of a shunt capacitor and frequency invariant susceptance interposed between a pair of admittance inverters, as shown in Fig. 5b. At this stage it is also convenient to incorporate the non-unity terminating resistance into one of the inverters.

Next, the series inductors and frequency invariant reactances within the pole sections are replaced as shown in Fig. 5c. The following identity is utilized:

$$\begin{bmatrix} 1 & sL + jX \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & j \\ j & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ sC + jB & 1 \end{bmatrix} \begin{bmatrix} 0 & -j \\ -j & 0 \end{bmatrix} \quad (20)$$

where

$$C = L \quad (21)$$

$$B = X \quad (22).$$

As noted by Levy [2], the exact equalities of (21) and (22) would be modified by a factor of M^2 if the immittance inverters were of immittance M rather than unity.

The circuit section between nodes 1 and 4 of Fig. 5c has the admittance matrix

$$[Y] = \begin{bmatrix} Y_{11} & -jM_{12} & 0 & 0 \\ -jM_{12} & Y_{22} & -j & -sC_{24} - jB_{24} \\ 0 & -j & Y_{33} & j \\ 0 & -sC_{24} - jB_{24} & j & Y_{44} \end{bmatrix} \quad (23)$$

where

$$Y_{11} = sC_1 + jB_1 \quad (24)$$

$$Y_{22} = s(C_2 + C_{24}) + j(B_2 + B_{24}) \quad (25)$$

$$Y_{33} = sC_3 + jB_3 \quad (26)$$

$$Y_{44} = s(C_4 + C_{24}) + j(B_4 + B_{24}) \quad (27).$$

Following the reduction process outlined by Levy [2] the admittance matrix from (23) is reduced to the form shown in (28).

Additional operations performed beyond Levy's [2] matrix reduction include multiplying elements of rows 1 and 3 as well as columns 1 and 3 by –1, yielding finally

$$[Y] = \begin{bmatrix} Y_{11} & j & 0 & jM_{14} \\ j & Y_{22} & jM_{23} & jM_{24} \\ 0 & jM_{23} & Y_{33} & j \\ jM_{14} & jM_{24} & j & Y_{44} \end{bmatrix} \quad (28)$$

where

$$Y_{11} = sC_1 + jB_1 \quad (29)$$

$$Y_{22} = s\frac{(C_2 + C_{24})}{M_{12}^2} + j\frac{(B_2 + B_{24})}{M_{12}^2} \quad (30)$$

$$Y_{33} = s\left(\frac{C_2 + C_{24}}{C_2}\right)^2 C_3 + j\left(\frac{C_2 + C_{24}}{C_2}\right)^2 B_3 \quad (31)$$

$$Y_{44} = sC_4'' + jB_4'' \quad (32)$$

$$C_4'' = \left(C_4 + \frac{C_2 C_{24}}{C_2 + C_{24}}\right) \quad (33)$$

$$B_4'' = \left(B_4 + \left(\frac{C_2}{C_2 + C_{24}}\right)^2 B_{24} + \left(\frac{C_{24}}{C_2 + C_{24}}\right)^2 B_2\right) \quad (34)$$

$$M_{24} = \left(\frac{C_2 B_{24} - B_2 C_{24}}{M_{12}(C_2 + C_{24})}\right) \quad (35)$$

$$M_{23} = \frac{(C_2 + C_{24})}{M_{12} C_2} \quad (36)$$

$$M_{14} = \frac{-M_{12} C_{24}}{(C_2 + C_{24})} \quad (37).$$

V. SUMMARY

A method for the direct synthesis of networks containing an arbitrary number of General Sections has been presented.

The technique is believed to represent a significant advance in the state of the art, in that it removes previous limitations of network order, and greatly simplifies the synthesis of lower order networks containing multiple General Sections.

REFERENCES

[1] 'General Prototype Network Synthesis Methods for Microwave Filters' – R.J. Cameron, ESA Journal 1982, Vol.6, pp. 193-206.

[2] 'Direct Synthesis of Cascaded Quadruplet (CQ) Filters' – Ralph Levy, IEEE Trans. Microwave Theory Tech., vol. MTT-43, pp. 2940-2944, December 1995.

[3] 'Filters with Single Transmission Zeros at Real or Imaginary Frequencies' – Ralph Levy, IEEE Trans. Microwave Theory Tech., vol MTT-24, pp. 172-181, April 1976.